Super-conservative interpretation of muon $g - 2$ results applied to supersymmetry

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Abstract

The recent developments in theory and experiment related to the anomalous magnetic moment of the muon are applied to supersymmetry. We follow a very cautious course, demanding that the supersymmetric contributions fit within five standard deviations of the difference between experiment and the standard model prediction. Arbitrarily small supersymmetric contributions are then allowed, so no upper bounds on superpartner masses result. Nevertheless, non-trivial exclusions are found. We characterize the substantial region of parameter space ruled out by this analysis that has not been probed by any previous experiment. We also discuss some implications of the results for forthcoming collider experiments.

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The relationship between the half-integral spin $\vec{s}$ of the muon and its magnetic moment $\vec{\mu}$ is written by convention as

$$\vec{\mu} = g\frac{e\hbar}{2m_\mu c}\vec{s} \equiv (1 + a_\mu)\frac{e\hbar}{m_\mu c}\vec{s}. \tag{1}$$

The non-zero value of a_μ accounts for radiative corrections to the semi-classical relationship. It is convenient for us to introduce a different variable directly related to a_μ:

$$\delta_\mu \equiv (a_\mu - 11659000 \times 10^{-10}) \times 10^{10}. \tag{2}$$

The current world average of the experimental measurement [1, 2] of δ_μ is

$$\delta_\mu^{\text{exp}} = 203 \pm 8. \tag{3}$$

The computation of the theoretical prediction for δ_μ in the Standard Model (SM) framework has been an impressive on-going effort by many groups [3]. A thorough analysis of this has been presented recently by Davier *et al.* [4]. They present two results, depending on whether they use τ decay data or e^+e^- collider decay as their primary source for understanding the hadronic loop corrections of the vacuum polarization diagrams contributing to the muon magnetic moment. The results are

$$\delta_\mu^{\text{SM}} = \begin{cases} 169.1 \pm 7.8 & (e^+e^- \text{ based}); \\ 186.3 \pm 7.1 & (\tau \text{ decay based}). \end{cases} \tag{4}$$

The systematic uncertainties differ in the two approaches, and the final results are in mild disagreement. This prompted Davier *et al.* to not combine the analyses.

The resulting difference between theory and experiment [4] is

$$\delta_\mu^{\text{exp}} - \delta_\mu^{\text{SM}} = \begin{cases} 33.9 \pm 11.2 & (e^+e^- \text{ based}); \\ 16.7 \pm 10.7 & (\tau \text{ decay based}). \end{cases} \tag{5}$$

Therefore, the current results indicate a quite tantalizing 3σ discrepancy [5] if one prefers the e^+e^- data, or a not-so-tantalizing 1.6σ discrepancy if one prefers the τ decay data.

While the τ decay based analysis does depend on significant theoretical input [6], in this letter we do not attempt to argue for one theoretical estimate over another, but rather wish to demonstrate that the muon $g-2$ measurement is interesting even if we take the most wide and conservative estimate of the theoretical and experimental uncertainties.

To accomplish this goal, we will take the union of the 5σ allowed regions of $\delta_\mu^{\text{exp}} - \delta_\mu^{\text{SM}}$ for the e^+e^- based approach and the τ decay based approach, and then declare that the supersymmetric contribution δ_μ^{susy} must fall into this range. Numerically, this works out to

$$-36.8 < \delta_\mu^{\text{susy}} < 89.9. \tag{6}$$

Note that the parameter space associated with $\delta_\mu^{\text{susy}} \simeq 0$ is in the allowed region. Extremely heavy superpartners will just decouple from the muon $g-2$ computation and yield a very small contribution, and so no upper bounds on superpartners can be stated in this analysis.

The immediate question now is whether this large allowed range has any impact on our view of supersymmetric parameter space given the myriad of other experiments that have been performed over the years, including collider physics direct searches for superpartners. Interestingly, the answer is yes. Especially at high $\tan\beta$, there are many combinations of smuon and chargino masses that are ruled out by this conservative muon $g-2$ analysis that have not been probed by any other experiment. Although it is impossible to succinctly characterize the complete parameter space that is excluded, below we will give several illustrations which convey the power of the muon $g-2$ experiment even under this most conservative approach to it.

Some might argue that taking a 5σ union for the allowed supersymmetry contribution is too conservative, and the muon $g-2$ experiment is more constraining than what will result from our analysis. That may well be, but our goal is to arrive at *exclusion* results that no reasonable person would quarrel with. In other words, there may be more supersymmetry parameter space excluded than we presented here, and perhaps the data is even telling us that some regions of supersymmetry space are being selected by the data. However, our emphasis here is that there is no reasonable chance that our declared excluded region can ever be resurrected by future data or analysis. Given some remaining skepticism about the SM theory computation (e.g., see ref. [7] regarding the light-by-light contribution), we feel our super-conservative approach to the data and theory is reasonable.

The recent body of work on supersymmetric corrections to muon $g-2$ is extensive [8, 9]. Our conventions for parameters and computation of the supersymmetric contributions to muon $g-2$ follow the details presented in ref. [10]. Our procedure is to compute the supersymmetric corrections as a function of the heavier smuon mass and the lighter chargino mass, under some basic restrictions that either simplify the presentation and are theoretically motivated, or insure that there is no conflict with other experiments. In all cases we require the following conditions be satisfied for parameters at the weak scale:

- All supersymmetry parameters such as μ, M_2, *etc.* are real (no CP violation effects, so the possibility [11] of an electric dipole moment does not arise).

- $|\mu| > M_2$, which is well within the expectations of minimal supersymmetry breaking schemes, and is typically valid for non-minimal scenarios discussed in the literature.

- $M_1 = 0.5 M_2$, which is required in simple gaugino mass unification scenarios.

- The scalar cubic coupling of smuons to the Higgs field satisfies $|A_\mu|/m_{\tilde{\mu}_2} < 3$, in the notation of [10], in order to avoid electric charge-violating vacua. (The particular value 3 chosen here has only a very mild impact on the results.)

- Smuon masses must be greater than 95 GeV to be consistent with LEP results [12].

The theoretical assumptions given above subsume a very large class of theoretical models for supersymmetry breaking. These include, but are not limited to, flavor-preserving minimal supergravity-inspired ("mSUGRA") models and minimal gauge-mediated supersymmetry breaking (GMSB) models. By making further assumptions, one can relate the magnitude of μ to the other parameters by requiring correct electroweak symmetry breaking within the confines of a particular model. However, there are many ways that these commonly made assumptions can be evaded by simple model extensions. Therefore, in keeping with our conservative approach to the data and the SM calculation, we prefer to maintain as general a model framework as possible. More particular assumptions of course lead to stronger exclusions. Again, the assumptions listed above are employed only to construct wieldy illustrations of the exclusions that the muon $g - 2$ experiment can impose.

Given our basic assumptions listed above, we show in fig. 1 the excluded area of the lighter chargino and heavier smuon plane, with different exclusion contours for various values of $\tan \beta$. In each case, everything below the exclusion contour is inconsistent with the muon $g - 2$ experiment and has never before been constrained by another experiment. We choose to make the plots using the heavier† smuon mass; the boundary of the allowed region is then obtained when the other smuon is not much lighter. As expected, the excluded region grows substantially with larger $\tan \beta$. Even for the relatively low value of $\tan \beta = 6$ there is still a significant portion of parameter space excluded by the $g - 2$ experiment. Graphically it looks like a rather small region in the lower left corner of the graph, but physically it excludes chargino masses as much as 80 GeV beyond the current limits, for low smuon masses. This is an important part of parameter space for Tevatron searches. The excluded region is smaller when we consider $\mu > 0$, as this is correlated with $\delta_\mu > 0$, which is less constrained than the $\delta_\mu < 0$ ($\mu < 0$) region. [The existence of small excluded regions that stubbornly persist as one looks further along the lighter chargino mass axis to the right is not related to $g - 2$, but rather the requirement that $m_{\tilde{\mu}} > 95\,\mathrm{GeV}$. For each $\tan \beta$, and given a lower limit on $|\mu| > M_2$, there is a minimum value

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†Although we do *not* assume it here, in many theoretical models the heavier smuon is mostly $\tilde{\mu}_L$, and plays a more significant role in collider phenomenology because of its greater coupling to charginos and neutralinos.
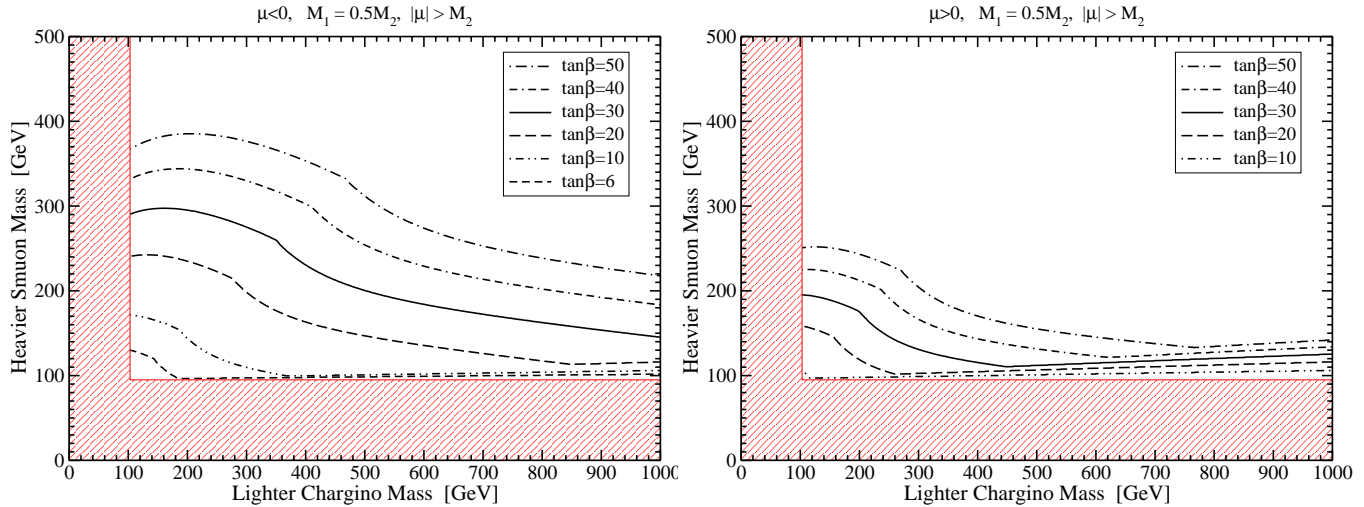
Figure 1: The region below each line is excluded by the muon $g - 2$ experiment, with the assumptions of gaugino mass unification, $|\mu| > M_2$, and $m_{\tilde{\mu}_1} > 95\,\text{GeV}$. The different lines correspond to different values of $\tan\beta = 50, 40, 30, 20, 10, 6$ from top to bottom. The left panel is for $\mu < 0$ and the right panel is for $\mu > 0$. The region constrained by direct searches at the LEP collider is shaded.

of level repulsion induced by the off-diagonal term of the smuon mass matrix $(m_\mu \mu \tan\beta)$ such that it is impossible to have both $m_{\tilde{\mu}_1} \simeq 95\,\text{GeV}$ and $m_{\tilde{\mu}_2} \simeq 95\,\text{GeV}$. Therefore, the heavier smuon mass must be a few GeV or more above $95\,\text{GeV}$.]

Let us now consider how the exclusions depend on the rather conservative assumptions we have made. The boundary of the excluded region is saturated by large (but not arbitrarily large) $|\mu|/M_2$ for lighter charginos, and by the minimum allowed value of $|\mu|$ for sufficiently heavy charginos. To illustrate this, we show in the left panel of fig. 2, for $\tan\beta = 30$ and $\mu < 0$, how the excluded region increases as one raises the minimum allowed value of $|\mu|/M_2$. (The solid line is the same as in fig. 1.) In many models of supersymmetry breaking, $|\mu|/M_2$ is required to be well above 1 in order to have correct electroweak symmetry breaking, but we see that increasing the minimum $|\mu|/M_2$ ratio only affects the exclusion contours for quite heavy charginos.

In the right panel of fig. 2, the dashed, dash-dotted and dash-dot-dotted lines are stronger exclusion contours that employ assumptions on the supersymmetric parameter space in addition to the ones discussed above (with $\tan\beta = 30$, $\mu < 0$, and $|\mu| > M_2$). For the dash-dot line, we add the requirement that $m_{\tilde{\tau}_1} > 80\,\text{GeV}$. Implicit in this is an assumption that the diagonal terms of the stau mass matrix are approximately the same as those of the smuon mass matrix, but the off diagonal terms are not ($m_\tau \mu \tan\beta$ rather than $m_\mu \mu \tan\beta$). In most fundamental
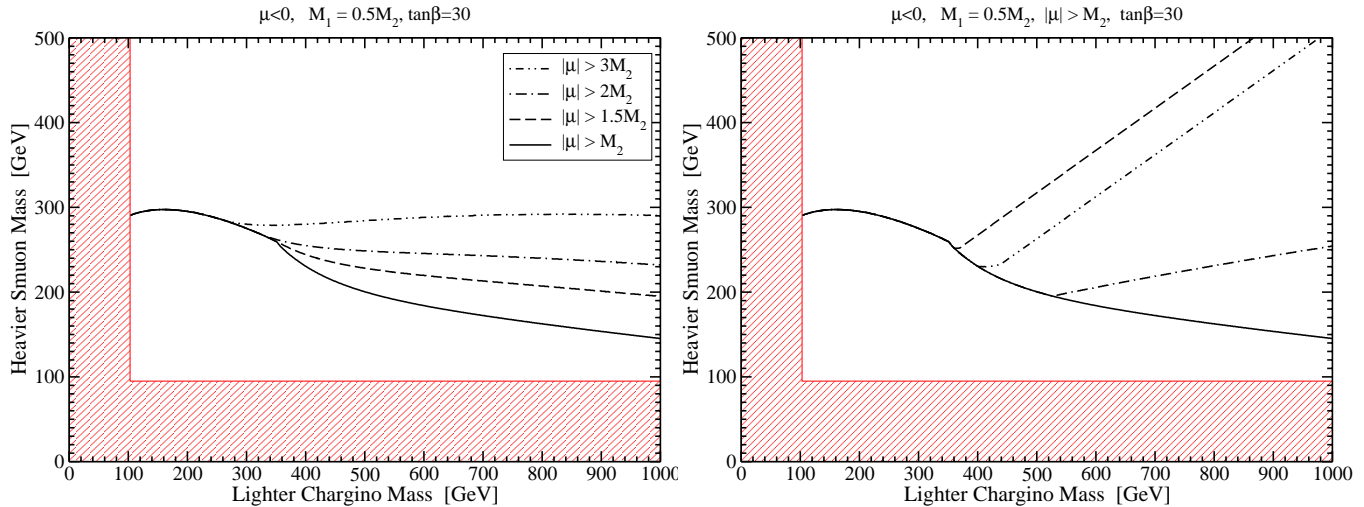
Figure 2: The impact of different assumptions is shown, for $\tan\beta = 30$ and $\mu < 0$. The regions under the contour lines are excluded by the $g-2$ experiment and the stated model assumptions. In the left panel, we show the excluded regions for different choices of the minimum allowed value of $|\mu|/M_2$. For lighter charginos, the exclusion is set by relatively large $|\mu|$, so the exclusion contours are not affected. In the right panel, we show the excluded areas obtained by adding the requirements (as described in the text) that $m_{\tilde{\tau}_1} > 80\,\text{GeV}$ (dash-dotted), $m_{\chi_1^0} < m_{\tilde{\mu}_1}$ (dash-dot-dotted), or $m_{\chi_1^0} < m_{\tilde{\tau}_1}$ (dashed). Notice that these additional constraints again have no effect if the chargino is not too heavy.

theories of supersymmetry breaking that approximately respect flavor, the diagonal terms of these mass matrices are the same at a high scale but diverge at the low scale where the mass eigenvalues need to be evaluated. The mismatch in diagonal entries between the smuon and stau mass matrices has a smaller effect than the off-diagonal mismatch on the difference in mass eigenvalues, so this approximation has more generality than might perhaps be naively expected. The excluded region is now bigger than that of the solid line, but the part of the boundary for lighter charginos is still entirely due to the muon $g-2$ result. The dashed line further requires that the lightest neutralino is the lightest superpartner, $m_{\chi_1^0} < m_{\tilde{\tau}_1}$. This is motivated by our desire to not have a charged lightest supersymmetric particle (LSP) in the event that R-parity is exactly conserved and the LSP is stable. Significant cosmological constraints apply to charged LSPs and it is unlikely that a stable, charged LSP can exist without modifying the standard scenario of cosmological history. The dash-dot-dot line in fig. 2b does not assume anything about the tau-slepton, and only requires that $m_{\chi_1^0} < m_{\tilde{\mu}_1}$. It is apparent that the additional constraints just mentioned have no effect on the exclusion curve when a chargino is light (in this case, lighter than about 360 GeV).

In fig. 3 we make the same plots as in fig. 1, but with the additional constraint that the
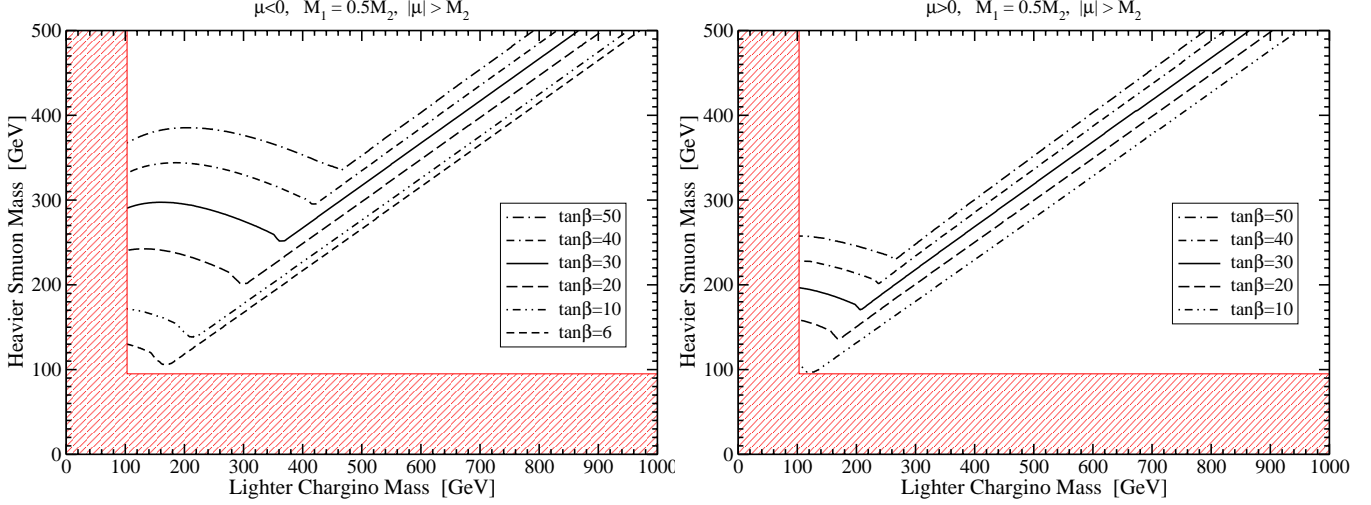
Figure 3: As in fig. 1, but with the additional constraint that a neutralino is the lightest supersymmetric particle: $m_{\chi_1^0} < m_{\tilde{\tau}_1}$. Here again, the regions under the contour lines are excluded by the $g-2$ experiment and the stated model assumptions. The left panel is for $\mu < 0$ and the right panel is for $\mu > 0$.

lightest supersymmetric particle should be a neutralino ($m_{\chi_1^0} < m_{\tilde{\tau}_1}$). The straight parts of the exclusion contours on the right part of each panel are set by the requirement that the lightest neutralino be the lightest superpartner (less than the stau mass). The remaining part of each exclusion contour curve is due to the muon $g-2$ experimental result. The region is significant and demonstrates the probing/exclusion capability of the muon $g-2$ experiment. Again, even for the relatively low value of $\tan\beta = 6$ there is still a significant portion of parameter space excluded by the $g-2$ experiment.

There are several interesting conclusions one can draw from figs. 1, 2 and 3 in addition to just demonstrating a rather large excluded area. For example, if $\tan\beta > 30$ and $\mu < 0$ and a chargino is found at or below $360\,\mathrm{GeV}$, there is no way that both smuons could have mass below $250\,\mathrm{GeV}$, given our assumptions. If we assume in addition that a neutralino is the lightest supersymmetric particle, then this lower bound on the heavier smuon mass becomes independent of the chargino mass. Heavy slepton mass scales such as this would make a $\sqrt{s} = 500$ GeV linear collider incapable of detecting both sleptons and measuring their masses and mixings. Likewise, if both smuons are found below about $240\,\mathrm{GeV}$, then a $\sqrt{s} = 500$ GeV linear collider should be able to discover them and study them with enough luminosity, but would not be able to discover and study even one chargino if $\tan\beta > 30$ and $\mu < 0$ with our assumptions.

Similar correlating statements apply to the Tevatron and LHC. For example, from figs. 66-

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69 of ref. [13] we see that the Tevatron upgrade will have a significant discovery capability for light charginos and neutralinos through the trilepton signal. For $\tan\beta$ larger than a few (which is suggested by the failure of LEP to detect a Higgs scalar boson), the parameter space in which a trilepton signal might be visible tends to divide into two disconnected regions: a region in which sleptons are comparable in mass to the charginos and neutralinos, and a region in which the sleptons are much heavier.

First, consider the case that sleptons are comparable in mass to the charginos and neutralinos. The sleptons generally greatly increase the branching fractions of chargino and neutralino decays into leptons, enhancing the clean (few backgrounds) trilepton signal. Masses of charginos and neutralinos up to nearly 200 GeV can be probed in these circumstances. However, for large $\tan\beta \gtrsim 10$, this is also the region where the $g - 2$ is very sensitive to supersymmetry. Therefore, for large $\tan\beta$ the future Tevatron trilepton search will have a strong (but not complete) overlap with the $g - 2$ exclusions we have found above, especially for $\mu < 0$. The muon $g - 2$ excluded region grows rapidly while the trilepton search sensitivity tends to fall quickly with increasing $\tan\beta$. So, in this region of parameter space under our assumptions the present muon $g - 2$ exclusions take a very large bite out of the otherwise new territory that the Tevatron will probe. The precise maximum values of $\tan\beta$ which the Tevatron can probe and which are not already ruled out by a conservative interpretation of the muon $g - 2$ results will of course depend strongly on the Tevatron experimental parameters which are still to be determined.

Considering instead the region of parameter space in which the sleptons are very massive, the charginos and neutralinos decay branching fractions to leptons asymptote, respectively, to the values of the W and Z decay branching fractions to leptons. This remains true provided the masses are light enough such that the second neutralino cannot decay into a Higgs boson plus lightest neutralino. In the extreme of all scalars decoupling, the chargino and second lightest neutralino can be discovered or excluded at the Tevatron with our assumptions if their masses are below about 130 GeV. In that region, the Tevatron has no competition from the present muon $g - 2$ result and is entirely complementary to it.

As for the LHC, there are circumstances in which Tevatron and $g - 2$ data would provide significant insights into LHC searches. For example, if charginos are found to be light at the Tevatron (e.g., mass $\lesssim 150\,\mathrm{GeV}$) and $\tan\beta$ is determined to be large (e.g., $\tan\beta \gtrsim 30$), the exclusion plots presented here imply that at least one slepton mass has to be greater than about $290\,\mathrm{GeV}$ ($190\,\mathrm{GeV}$) for $\mu < 0$ ($\mu > 0$). Sleptons with mass greater than about 300 GeV would be very challenging to directly detect at the LHC [14]. Or, if $\tan\beta$ has not been measured by the time LHC collects data, discovery of light sleptons and a light chargino with

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masses less than about 190 GeV would rule out large $\tan \beta > 30$ supersymmetry, for either sign of μ, because of its incompatibility with the muon $g - 2$ experimental result.

Our central point is that even the most conservative view of the data produces a large region of supersymmetry excluded only by the muon $g-2$, and no other experiment. The implications have the potential to be very important for future experiments. As we detailed in our previous paper [10], the constraints from $B(b \to s\gamma)$, Higgs mass, and relic abundance are not necessarily correlated in any meaningful way with $g - 2$, and so the $g - 2$ experiment is firmly established as an independently powerful probe of supersymmetry.

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